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AUTHOR: Henderson, Ronald W.; Landesman, Edward M.
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ABSTRACT

This report explores the student background characteristics that might be associated with success or failure in calculus and evaluates the effectiveness of remedial mathematics education. The sample consisted of two groups at the University of California, Santa Cruz: (1) all students (105) who took first quarter calculus in spring, 1985; and (2) all students (138) who enrolled in a remedial course during 1983-84 and 1984-85 academic years. A path analytic approach was used to evaluate the background data of sex, ethnicity, and high school grade point average (HSGPA); scores on the mathematics section of the Scholastic Aptitude Test; and performance in the calculus course. Neither the path from sex to HSGPA nor the path from ethnicity to HSGPA was found to be significant. The path from sex to Scholastic Aptitude Test was highly significant. The path from ethnicity to performance in calculus (CALCTOT) was not significant. The highest path coefficient in the model was from HSGPA to CALCTOT. A survey and interviews with a small sample also gathered information on study habits, use of support services, perception of adquacy of instruction, self-efficacy, and learning modality preference. Larger samples are needed to determine if the survey instrument might be a useful predictor of student mathematics performance. Survey instruments are appended. (BAE)

Research Group in Mathematics
and Science Education

A Preliminary Evaluation of Student Preparation for the Study of Calculus

Ronald W. Henderson

and

Edward M. Landesman

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A Preliminary Evaluation of Student Preparation for the Study of Calculus

The project reported in this document was undertaken in response to an invitation to submit a proposal for support to evaluate remedial education in mathematics at the Santa Cruz campus of the University of California. The invitation was issued by the Office of Student Preparation of the University of California, which functioned as an administrative channel for funding provided by a grant from the Fund for the Improvement of Postsecondary Education to the California Post Secondary Education Commission. The timing of the invitation coincided with our interest in looking into possible reasons for the high rate of failure among students enrolled in the first quarter of the calculus sequence, Math 11A, during the spring quarter, 1985.

It has been generally assumed that students who enter the University with weak backgrounds in mathematics enroll in calculus in the spring, because during the fall and winter quarters they have been busy taking remedial work to prepare themselves. We were interested in exploring the patterns of student background characteristics that might be associated with success or failure in calculus, as well as the effectiveness of remedial offerings. The spring, 1985 course was of special interest because the teacher had kept complete records of the students.

Given the expectation that former remedial students might be well represented in the spring 1985 calculus course, and the knowledge that detailed records on performance in this class were available to serve as criterion measures, it seemed reasonable to link our inquiry into possible reasons for failure in Mathematics 11a with the evaluation of remedial mathematics offerings. Under the best of circumstances, an evaluation undertaken after students have completed the remedial courses of interest would be limited to retrospective exploration. For it to be otherwise would require the study of students currently enrolled during the 1985-86 academic year, with follow-up data being gathered during the subsequent year. Funding provisions did not permit such an approach.

Considering the circumstances that precluded a detailed, comprehensive evaluation of remedial mathematics courses at this time, the present evaluation effort was designed, in large measure, to provide exploratory analyses that could establish a conceptual basis for an evaluation model for use in subsequent efforts. We therefore decided to focus our efforts on those students who were enrolled in Mathematics 11A (first quarter calculus) during the spring quarter, 1985, and on those students who had taken remedial mathematics on the UCSC campus either during the fall or spring quarters of that academic year, or at any time during the previous academic year.

Method

Subjects

The sample consisted of two overlapping groups of students (n = 238). The first subsample included all students who took

Math 11A (first quarter calculus) during the spring quarter, 1985, and for whom registrar data were available. (*n* = 105). This sample was of special interest because a high proportion (approximately 40 percent) of the class failed to pass. The second subsample was comprised of all students (subject to availability of data) who enrolled in a remedial course at UCSC during the 1983-84 and 1984-85 academic years (*n* = 138).

As previously mentioned, the spring quarter calculus course usually includes a higher proportion of students who have been unprepared to enter the calculus sequence upon entering the University in the fall quarter. We reasoned that during the fall and winter quarters, a number of these students may have been involved in remedial coursework[1] or in precalculus, in preparation for the calculus course offered during the spring. Our hope, therefore, was that a number of students who had taken remedial courses would also turn up on the roster of the spring 1985 calculus course. If this were the case, interval scale data (midterms and final examinations) would be available to serve as criterion scores. Such data would be more sensitive to variations in student performance than the pass/no-pass information available from registrar records. The degree of overlap (*n* = 5) found, once the data were analyzed, was much smaller than anticipated. Only five students from the spring, 1985 calculus course had previously taken a remedial course at UCSC.

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[1]For purposes of this report, remedial mathematics refers to courses in basic mathematics. Precalculus is not a remedial course.

Procedures

Electronic data files on the subsamples were obtained from the Office of the Registrar. Because funds to conduct the evaluation came late in the year, and the timing coincided with an unusually heavy workload for personnel in the Data/Information Systems department of the Registrar's Office, data were not received until well into the spring quarter. Once data files were received, a considerable amount of time was spent making conversions that were required in order for the data to be processed with standard statistical software.

Data on a large number of variables were made available to us. These data included information of background variables, such as the high school from which the student graduated, periods of probation during residency at UCSC, admission status, and so on. Where available, data were also provided on high school grade point averages, scores on the verbal and mathematics portions of the Scholastic Aptitude Test, and scores in various topic areas of the College Entrance Examination. Survey and interview data, obtained with the instruments described below, together with midterm and final examination scores for the calculus students, were merged with the registrar data files.

Instruments

Two instruments were developed for use in this evaluation: A "Math Learning Survey" and an interview schedule. These instruments are presented in Appendices A and B, respectively. Both instruments were developed largely on the basis of informal interviews with undergraduate students (not in the study sample) about their own study habits and perceptions of obstacles to learning mathematics. The information gathered also included these student's observations of study habits and impressions of barriers to mathematics learning among their peers. We also took our lead from literature on differences in response to failure experiences among learners with helpless versus mastery oriented approaches (Dweck, 1975; Henderson, 1982, in press).

Several categories of information to be gathered by survey and interview techniques were derived from this information. They included: study habits, use of support services, perceptions of adequacy of instruction, self-efficacy, and learning modality preference. Survey items were created to elicit information for each of these categories. Items pertaining to study habits included, for example, questions about the frequency of attending lectures, completion of homework, use of small versus large blocks of time for study, and so on. Questions regarding the use of support services included formal support services, such as tutors, and informal support, such as study with classmates. Perceptions of the adequacy of instruction dealt with textbooks (do they make unwarranted assumptions about precursor skills and concepts in the learner's repertoire?), teachers, and tests (are they written with sufficient clarity that the student understands the task presented?). Items intended to elicit information on mathematics self-efficacy were both direct (e.g., perceptions of ability to do well in mathematics) and indirect (e.g., reactions to frustration and failure in mathematics). Cognitive modality questions overlapped with the study habits category, but basically probed whether the student depended primarily on explanation to learn problem solving, or preferred visualization techniques.

The survey employed a question format based on Harter's (1985) approach. Each item consisted of two contrasting statements (not necessarily opposites). For each item, subjects chose the statement that was most true for themselves. Harter developed this format in her research on social competence to overcome some of the tendency of items in traditional self-concept scales to elicit socially desirable responses. Harter has employed this format successfully in various scales designed for use with subjects ranging from early elementary through college age samples.

Considering the small number of Math 11A students who had taken remedial courses at UCSC, it was decided that the survey should be mailed to all students from the spring, 1985 Math 11A class who had taken precalculus on the UCSC campus during a prior term and for whom both addresses and registrar records were available. The letter that accompanied the survey form is provided in Appendix C. The sample included the four remedial students who had taken both the target calculus course, and precalculus at UCSC. During interviews, we learned that some students had taken remedial, or pre-college mathematics at a community college before coming to UCSC. For reasons mentioned earlier, the survey was not mailed until about a week before final examinations.

The interview schedule was developed to probe for richer detail than the survey could provide, and to obtain background information about the students' mathematical experience. The questions fell into five categories: math history, math class information, study habits, math self-efficacy, and other. A focused interview approach was used, in which principal questions were followed by probes. The probes were used, as appropriate, to obtain desired information if the main questions did not elicit informative responses. Typical questions from the five categories included queries such as: "How did you feel about math as you were going through elementary and high school?", "What math classes did you plan to take when you came to the University?", "Tell me something about how you study for math.", Would you say that math has come easily to you, or have you had to struggle with it?", and finally, "How could your UCSC math preparation for calculus have been improved?".

The interview was administered to a sub-sample of the subjects who responded to the survey. All of the questions were administered during oral interviews which had been scheduled in advance. The students being interviewed were unaware of the questions that were going to be asked, but knew that the inquiry would be related to their mathematical experiences.

The interviews were scheduled by two advanced undergraduate psychology students and a graduate student in education, under the supervision of one of the investigators. These same students conducted the interviews and took notes on the responses.

Follow-up telephone calls were made to all students for whom numbers were available to inquire if the survey had been received, and to urge that it be completed and returned promptly. In some cases, students who had not responded to the mailed questionnaire responded to the survey items on the telephone. Student interviewers practiced role-playing the techniques for obtaining the survey data by telephone. None of the former remedial students responded to the survey, and when contacted by telephone to arrange for an interview, none felt they could spare the time during the hectic week and a half before final exams.

Results

A path analytic approach was employed to examine the effects of student individual characteristics and academic achievement variables on performance in calculus. These analyses were performed using only those students for whom all data were available for all variables to be included in the analysis.

Of the background data available from the Registrar's records, only those variables of greatest theoretical interest were included in the analyses. The dependent variable was performance in the calculus course (CALCTOT). The independent variables selected were sex and ethnicity, with high school grade point average (HSGPA) and scores from the mathematics section of the Scholastic Aptitude Test (SATMTH) serving as mediating variables. Initially, mathematics scores from the College Entrance Examination were of interest, but preliminary analyses revealed that the addition of this variable to a regression equation did not improve the prediction of calculus performance, and its inclusion would also have reduced the total number of students available for the analyses.

The Lisrel VI (Joreskog & Sorbom, 1984) computer program for structural equation modeling was used to compute path coefficients for the "untrimmed," or just-identified model displayed in Figure 1. The just-identified model (Pedhazur, 1982, p. 296) contained all possible paths and, as would be expected, reproduced the correlation matrix perfectly. The fit of the model was tested by the chi-square and root-mean-square

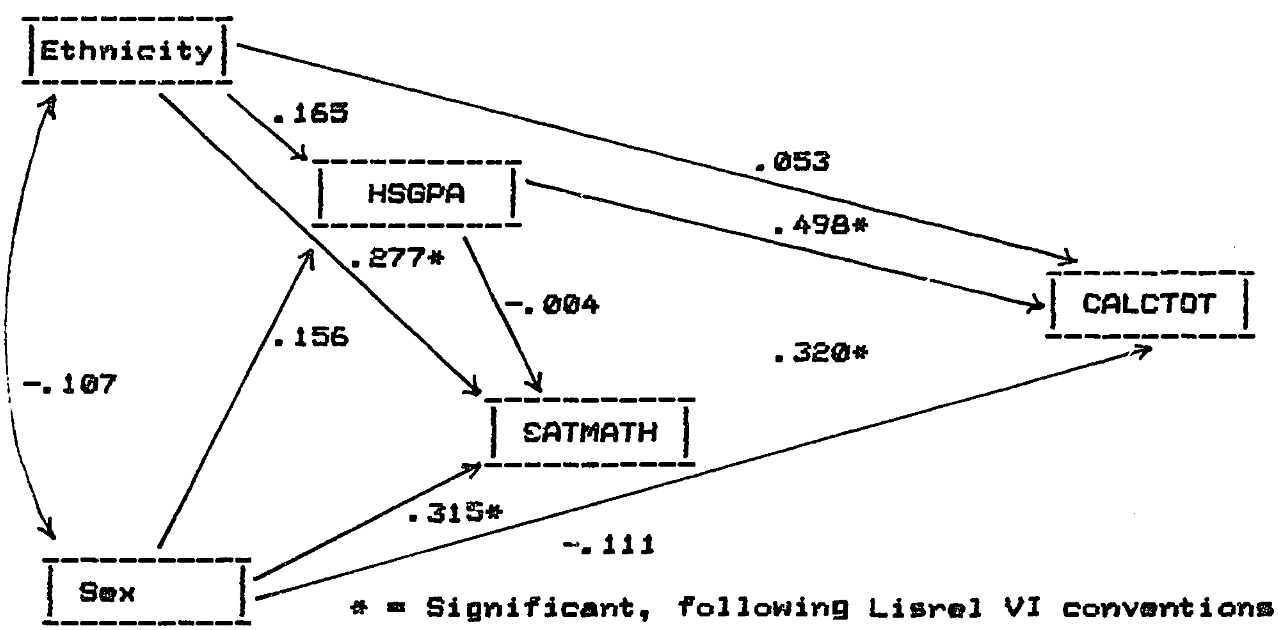

Figure 1: Untrimmed path model for influences on performance in calculus

methods. For the untrimmed model, the fit was perfect, with a chi-square and root-mean-square residual of 0. This, of course, is what one would expect with an untrimmed model. Following the conventions of Lisrel VI, paths with t values of 2 or greater

Table 1

Correlation and Reproduced Matrices

Variable	1	2	3	4	5
1. Ethnicity	-	-.107	.180	.264	.065
2. Sex	-.107	-	.079	.248	-.117
3. HSGPA	.155	-.056	-	.228	.489
4. SATMTH	.247	.248	.218	-	.301
5. CALCTOT	.103	-.177	.481	.286	-

Note: Intercorrelations are shown above the diagonal. Correlations reproduced from the trimmed model are below the diagonal. N=60.

were considered significant. The intercorrelations among the variables of the untrimmed or just-identified model are shown above the diagonal in Table 1.

Figure 2 presents the path diagram for the overidentified ("trimmed") model in which two of the original paths were blocked to represent a theoretical model (Pedhazur, 1982, p. 605).

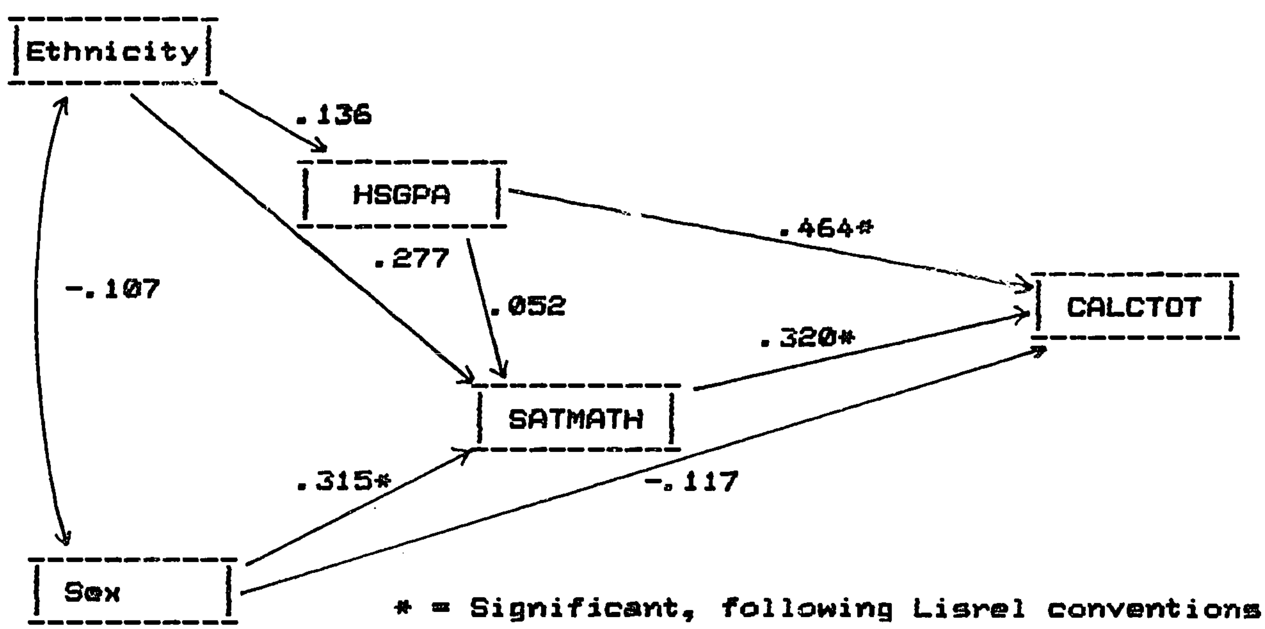

Figure 2: Trimmed path model for influences on performance in calculus

The reproduced correlations are displayed below the diagonal in Table 1. The path from ethnicity to CALCTOT was blocked because it was anticipated that most of the variation in calculus performance among students from underrepresented and non-underrepresented ethnic groups would be accounted for by indirect paths through HSGPA and SATMTH. The path from sex to HSGPA was blocked because we anticipated that sex would be unrelated to HSGPA but negatively associated with SATMTH. The paths from ethnicity to HSGPA and SATMTH were expected to carry negative values. goodness of fit test for the model (Chi-Square with 2 degrees of freedom = 1.88, p = .391; Root mean square residual = .039) indicated that the model could not be rejected. The Goodness of Fit Index was .988, and the Adjusted Goodness of Fit Index was .907.

The path coefficients for total effects for the XY relations for both the trimmed and untrimmed models are displayed in Table 2. The Mintab computer program (Pennsylvania State University, 1981) was used to compute the indirect paths. All indirect path coefficients were negligible, disconfirming the expectation that ethnicity would influence calculus performance indirectly through HSGPA and SATMATH.

The direct path from sex to CALCTOT was negative, suggesting a slight tendency for women in this particular calculus course to outperform the men. However, the path coefficient failed to achieve significance. Contrary to original expectations, the path from ethnicity to HSGPA was also non-significant. HSGPA had a direct and independent effect on CALCTOT. Possible exogenous

Table 2

Total Effects of X on Y and Y on Y for Untrimmed and Trimmed Path Models

	Sex	Ethnicity	SATMTH	HSGPA
CALCTOT				
Untrimmed	-.11	.05	.32	.50
Trimmed	-.12	*	.32	.48
SATMTH				
Untrimmed	.28	.29	–	.00
Trimmed	.28	.28	–	.05
HSGPA				
Untrimmed	-.04	.19	.00	–
Trimmed	*	.15	.05	–

Note: * indicates blocked path.

variables that might contribute to that influence were not included in the model.

The residuals, presented in Table 3, reflect the difference between the matrix for the original path model and the reproduced matrix based on the trimmed models. The generally small values for the residuals indicates a good fit for the model, although the residual for sex x HSGPA (.135) is a bit higher than would be desired.

Table 3

Fitted Residuals

	CALCTOT	SATMTH	HSGPA	SEX
CALCTOT	.00			
SATMTH	.02	.01		
HSGPA	.01	.01	.01	
SEX	.00	.00	.14	.00
ETH	.05	.02	.03	.00

Table 4

Descriptive Statistics for High School Grade Point Average and Mathematics by Remedial versus Non-Remedial

Variable	N	Mean	SD	Max	Min
HSGPA					
Remedial	61	3.06	63.13	4.00	33
Non-Remedial	63	3.40	27.53	4.00	256
SATMATH					
Remedial	66	445.91	85.79	610	280
Non-Remedial	70	542.57	88.93	750	310

For the entire sample of cases available from registrar records, students who took remedial mathematics at UCSC were compared with those first quarter calculus students who had not received remedial instruction. Descriptive statistics for this comparison are presented in Table 4. The mean HSGPA for the latter group of students for whom data were available ($n = 63$) was 3.40, whereas the average high school grade point average for remedial students was 3.06. Grade point averages were much more variable for remedial ($sd = 63.13$) than for non remedial students ($sd = 27.53$).

Cross tabulations were conducted for the categories <u>group</u> (remedial, non-remedial) by <u>course_performance</u> (pass, no-pass) for both precalculus and first quarter calculus. In the entire data set provided by the registrar, only four Math 11A students who had previously taken both a remedial course and precalculus were located. Three of those students passed and one failed

Table 5

Contingency Table for Remedial/Non-Remedial Groups by Dependent Variable Calculus Performance

Group	Calculus Performance	
	Fail	Pass
Remedial	*fo* = 4 (3.88%) *fe* = 1.84	*fo* = 1 (.97%) *fe* = 3.18
Non-Remedial	*fo* = 34 (33.01%) *fe* = 36.16	*fo* = 64 (62.14%) *fe* = 61.84

Chi Square (*df* 1) = 4.194, *p* (.038

Contingency Coefficient = .198

calculus. Of the 98 non-remedial students, 34 (35 percent) failed and 64 (65 percent) passed. The contingency table and the chi square statistic for these frequencies are presented in Table 5.

Chi square was significant, indicating that the pass/no-pass proportions for the two groups did differ from chance expectancies. Although chi square was significant, the contingency coefficient of .198 suggests that, from a practical point of view, the amount of variance in calculus performance accounted for by remedial group membership was quite small (less than 4%).

Of a total of 33 non-remedial students who took precalculus before enrolling in the spring, 1985 calculus class, 4 (12%) failed precalculus, while 29 (88%) passed it. Of the four former remedial students, one failed precalculus, and three (75%) passed

Table 6

Contingency Table for Remedial/Non-Remedial groups by Dependent Variable Precalculus Performance

Group	Precalculus Performance	
	Fail	Pass
Remedial	fo = 1 (3%) fe = .54	fo = 3 (8%) fe = 3.46
Non-Remedial	fo = 4 (11%) fe = 4.46	fo = 29 (78%) fe = 28.54

Chi Square (*df* 1) = .506, *ns*

Contingency Coefficient = .18

it. A chi square test conducted on the group (remedial, non-remedial) by precalculus performance (pass, no-pass) crossbreak was not significant (chi square = .506, *ns*) indicating that the actual frequency of passing/not passing for the two groups did not differ from the frequency expected by chance. These data are displayed in Table 6.

Next, a chi square test was conducted to explore the relation between performance in precalculus (pass/fail) and successful completion of calculus in a subsequent quarter. The contingency table for this analysis is shown in Table 7. The observed frequencies did not diverge from expected frequencies in any cell, resulting in a non-significant overall chi square.

Table 7

Contingency Table for Precalculus Performance by Dependent Variable Calculus Performance

Precalculus Performance	Calculus Performance: Fail	Calculus Performance: Pass
Fail	fo = 2 (5%) fe = 2.53	fo = 4 (11%) fe = 3.47
Pass	fo = 14 (37%) fe = 13.47	fo = 18 (47%) fe = 18.53

Chi Square (df 1) = .225, ns
Contingency Coefficient = .077

We next addressed the question of whether students enrolled in precalculus during one term were more or less successful than students enrolled in another term, with a different instructor. Table 8 presents the contingency table for that analysis. The non-significant chi square (2.105) suggests that the term of enrollment in precalculus was unrelated to later performance in calculus, as represented by a pass/fail score (contingency coefficient = .229).

The maximum total score obtained by any nonremedial student for the calculus course was 399 (of a possible 400 points). The highest score achieved by any student with a background of remedial coursework in mathematics at UCSC was 258, while the next highest score for this group was 134. Five students who took the first quarter calculus course during the spring quarter,

Table 8

Contingency Table for Term Precalculus Taken by Dependent Variable Calculus Performance

Term	Calculus Performance	
	Fail	Pass
Spring '84	f_o = 1 (3%) f_e = .84	f_o = 1 (3%) f_e = 1.16
Spring '84	f_o = 3 (8%) f_e = 5.05	f_o = 9 (24%) f_e = 6.95
Winter '85	f_o = 12 (32%) f_e = 9.73	f_o = 12 (32%) f_e = 14.27

Chi Square (*df* 2) = 2.105, *ns*

Contingency Coefficient = .229

1985, had previously taken at least one remedial mathematics course on the UCSC campus. Of these five, only one passed the calculus course, as indicated in data presented above. Given that situation, we decided to take a closer look at data for the remedial students who later participated in the calculus course. Four of the five had been enrolled previously in precalculus. One former remedial student failed both precalculus and calculus. This student had a mathematics SAT score of 330, and no high school grade point average was available. The remaining three students who had taken precalculus passed it, but subsequently failed calculus. For these students, HSGPAs were 3.20, 2.38, and 3.82, respectively. SATMATHs for the first two of these students were 400 and 510, respectively, with no score available for the third. The remedial student who passed calculus did not take the precalculus course at UCSC, had a HSGPA of 3.82, and no SATMATH score.

The fact that so few students who had taken remedial mathematics appeared in the calculus course prompted us to wonder what happened to these students following their remedial course work. The assumption had been that many students take remedial mathematics to prepare themselves to go on to higher level courses in mathematics. To address this question we obtained record cards, current through the spring quarter, 1986, for all students who had taken any remedial mathematics course during the 1983-84 and 1984-85 academic years. Student records were then classified on the basis of the kinds of quantitative courses taken during residency at UCSC. Non-remedial courses offered by the Board of Studies in Mathematics constituted one category. Categories were also formed for quantitative courses in the Natural Sciences (exclusive of mathematics courses) and in the Social Sciences. Those courses that were designated as meeting the quantitative course criteria for campus general education requirements were included in this count. The percentages of students taking courses in each category are presented in Table 9.

During the 1983-84 academic year, remedial courses could be used to satisfy the quantitative general education requirement of the campus. Although this was no longer true in 1984-85, the difference in the proportion of students who took additional

Table 9

Percent of Remedial Math Students Taking Quantitative Courses

			CATEGORY					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	
Year	Math	Nat Sci	Soc Sci	1 + 2	1 + 3	2 + 3	1+2+3	Total
83-84	9	10	14	13	2	0	1	49
84-85	12	15	11	10	1	2	4	55

Note: Category 1 = 1 or more mathematics course, 2 = 1 or more quantitative Natural Science Division class, 3 = 1 or more quantitative Social Sciences Division class, 4 = categories 1 + 2, 5 = categories 1 + 3, 6 = categories 2 + 3, 7 = categories 1 + 2 + 3.

quantitative course work does not appear to differ markedly for the two cohorts. In 1983-84, 49% of the students who took one or more remedial mathematics courses took at least one additional quantitative course during the subsequent two years. For 1984-85, 55% of these students went on to take at least one quantitative course during the ensuing academic year. Since the 1984-85 cohort will not be able to apply the remedial course work to the quantitative requirement, it does seem that many (almost half) of these students are postponing course work to meet this requirement.

The survey instrument that was developed to obtain information on attitudes toward mathematics, study habits, and perceptions regarding instruction and instructional materials was subjected to a reliability analysis. The internal consistency

Table 10

Item Reliability Data for Mathematics Learning Survey

Item	Scale Mean If Item	Scale Variance If Item	Corrected Item-Total	Alpha If Item Deleted
MMCLS	66.61	61.08	.46	.67
TUTR	67.56	67.79	-.14	.72
RTXT	67.00	59.18	.38	.67
HMWRK	66.44	63.09	.20	.69
STCLM	66.33	68.35	-.18	.71
TSKP	66.28	61.62	.36	.67
SB1ST	67.83	61.79	.31	.68
LRNAB	67.28	58.21	.59	.65
SYNOT	67.61	57.66	.58	.65
TEXP	66.89	59.16	.41	.67
SSAC	67.78	63.48	.15	.68
GVUP	66.61	60.49	.37	.67
HEREX	66.50	61.44	.21	.69
BBST	66.78	67.83	-.14	.72
MTSH	67.44	55.09	.67	.64
TXSS	67.33	58.71	.46	.66
CWOS	68.11	63.75	.13	.69
WKEXM	67.11	61.75	.25	.68
VISAB	67.61	58.96	.41	.67
TSSX	66.33	63.41	.18	.69
SUPEX	67.56	55.79	.67	.64
VSNTS	66.56	75.91	-.61	.75
ETHT	66.17	64.62	.25	.68
SKFO	67.00	65.18	.00	.71
BTQT	66.94	58.17	.60	.65

Note: MMCLS = misses math classes; TUTR - uses math tutors; RTXT = reads every assigned chapter; HMWRD = does all homework; STCM = studies with classmates; TXSKP = feels texts skip information; SBTST = studies only before tests; LRNAB = believes has natural math ability; SYNOT = feels intimidated by symbols and notation; TEXP = thinks teachers' explanations adequate; SSAC = studies soon after class; GVUP = gives up easily; HEREX = need to hear explanation; BBST = uses big blocks of time for study; MTSH = wording on tests hard to understand; TXSS = thinks texts skip steps; GWOS = does well with little studying; WKEXM = works examples from text; VISAB = can visualize abstract concepts; TSSX = learns best from teacher of same sex; SUPEX = understands peers' explanations best; VSNTS = uses notes for study; ETHT = ethnicity of teacher irrelevant; SKFO = sketches to visualize problem; problem; BTQT = quits trying after poor test results.

reliability, as determined by the standardized item alpha.

Two subsequent reliability analyses were then conducted. All items with negative item-total correlations were eliminated from the first of these analyses, resulting in a scale of 20 items. For the second analysis, all items with item-total correlations of less than .20 were eliminated. Elimin..tion of the five items that were correlated negatively with . e total score improved the coefficient alpha from .72 to a) respectable reliability of .83. The elimination of an additio.al three items that correlated with the total score at levels below .20 did not improve the reliability of the scale.

The response frequencies for survey items are presented in Appendix D. The directionality of items was randomized in the actual survey (see Appendix A), but is kept constant in the tabulation presented in Appendix D, for clarity of presentation. The results are interesting from a descriptive point of view. Most of the students who responded to the survey reported relatively good study habits. They attended class regularly and were conscientious about doing their homework and reading assignments. Nevertheless, there was a wide range of variation in whether they tended to do most of their studying just before a test, or reviewed on a regular basis. Most (79%) felt they needed to spend a lot of time studying in order to do well in mathematics, but they (74%) also admitted a tendency to put off studying math for as long as possible.

The majority of these students expressed a degree of confidence that they had a natural mathematics ability, although a few (30%) expressed some degree of doubt that they could expect to learn mathematics well. Even among this relatively successful group of students, the majority (57%) confessed to feeling intimidated by mathematical symbols and notation.

Students were almost unanimous in saying that they learn best by seeing an example, as contrasted with listening to explanations. Yet, only half of those who responded felt they could visualize concepts well.

A substantial minority (48%) found the wording of tests hard to understand, and the same proportion felt that textbooks leave out important information. Far fewer of these students (29%) thought teachers skipped important steps in their presentations.

Based on self-report, most of the students who responded to the survey would appear to be what Dweck (1975) has referred to as mastery oriented students. In the face of difficulties in solving mathematics problems, most of this group did not simply give up in frustration. Rather, they were likely to seek help. And when they performed poorly on a test, their most likely response was to try harder next time. Such a response contrasts with that of students who have acquired a pattern of "learned helplessness," and who tend to respond by attributing their failure to a lack of ability, and to reduce their subsequent level of effort (Henderson, 1981).

From these descriptive data, and from the psychometric data presented in Table 10, we would conclude that the survey does seem to be tapping variations in students' approaches to the study of mathematics and their perceptions of instruction in the subject. The reliability analyses indicates that the survey has reasonably sound psychometric qualities, but a regression analysis, with CALCTOT as the dependent variable, failed to provide evidence that the instrument might serve as a useful predictor of student performance in mathematics. It should be noted, however, that the students for whom survey results were available had generally not had substantial difficulties in their mathematics learning histories. Adequate validity studies would require a much larger sample, and must include students who experienced difficulties with mathematics instruction.

As previously noted, only a very small sample of students were interviewed ($\underline{n}$ = 7). Even though these students had not received remedial instruction in mathematics at UCSC, all had taken precalculus prior to calculus. The responses to some of the interview items are worthy of note because they may be indicative of difficulties faced even by students who are not the most seriously deficient in mathematics preparation upon entry to the University.

In response to the question "How well did it (math in high school) prepare you for math at the University," about one half of the students felt that they were not prepared to take mathematics at the University of California. In response to questions dealing with math class information, it was interesting to note that some students mentioned how manipulating a formula is easy, but is not retained. Visualizing a problem is difficult, lasts forever, and builds understanding. Another interesting comment was that instructors omit steps that should be included, or, as one student said, "They assume you know a lot".

In answering the question, "Would you say that math has come easily to you, or have you had to struggle with it?", about one half stated that they had to work at it. Typical comments were, "Math has always been a struggle.", "It doesn't come easily.", and "It used to be easy, until trigonometry during the junior year in high school.

Of the one half who felt that math came easily, comments included such statements as: "Reasonably. With a lot of time put in anything is easy.", "I have always been strong in math, but I think kids with calculus in high school probably did better than me.", "Math has always come easy."

Interestingly enough, in response to the question, "Do you like doing math?", there was no direct correlation between those who found math easy, and those who enjoyed the subject, nor was there any agreement among those who found math difficult and disliked the subject.

Finally, it is interesting to note the response of students when asked, "How could your UCSC math preparation for calculus have been improved? Responses ranged from "I don't think that people with a strong background in math need to take precalc. It is people like me who are very anxious about math who take it and get nothing from it, because it reviews the math they can already do. ... I didn't like the teacher in my first calculus class. He was slow. But I liked the second one. Liking the teacher helps, and I think that the teachers in high school were better. Oh yes, I don't think anyone understands word problems."

In response to the same question, another person responded that "The University should offer all levels of math at UCSC, since many people haven't had math for many years, and need refreshers." Another student said, "In regards to precalc, I think that the teacher could have, and should have, presented a better understanding of the concepts. There should be less submersion into formulas, and more teaching of how to visualize the problems."

Discussion

A path analysis was conducted to serve as the basis for the design of an evaluation model capable of taking student background and characteristics into account. A number of variables of potential interest, such as special admission, or high school of graduation, could not be included in the model because the numbers of students in each category was too small for this kind of analysis. With a sufficient data base, other variables of interest could have been factor analyzed to yield latent variables for use in the model. Even though this could not be done for the present analysis, the results have several interesting features.

It must be cautioned that the small number of remedial students involved in the analysis does not allow generalization of the specific results of this study, even to the remedial courses offered at UCSC during the period of time under study.

As expected, the path from sex to HSGPA was not significant, but given the expectation that, in general, underrepresented students would be more heavily represented than non-underrepresented students among those coming to the University with weaker academic preparation, we were surprised that the path from ethnicity to HSGPA was not significant. We have no ready explanation for this finding. If it is true that underrepresented students, in general, have less opportunity than majority students to receive adequate preparation for college, then we would expect lower grade point averages among these students. The exception would be if they came from ethnically homogeneous high schools, and if high school grades represent their status within the norm group of their own particular school rather than a criterion-referenced indicator of course mastery. This seems a plausible explanation, given the highly significant path from ethnicity to SATMATH, for which the norm group is national.

The path from sex to SATMATH was also highly significant. The positive direction of the path coefficient suggests that women students tended to score lower than men on the mathematics portion of the SAT, a finding that is congruent with national data (Jensen, 1980). The path from HSGPA to SATMATH was also significant.

The final dependent variable in the path model is CALCTOT. It is especially interesting that the direct path from ethnicity to performance in calculus was not significant, and it does not appear to be mediated by other variables in the model. We would emphasize that, in this discussion, the term "influence" is used in a statistical rather than a causal sense. Sex exerts its influence directly on SATMATH, which, in turn, influences performance in calculus.

The highest path coefficient in the model is the one from HSGPA to CALCTOT. The antecedents of HSGPA are not evident in this model.

Path analysis provides graphic means of studying hypothesized direct and indirect influences of variables on each other and on dependent variables. It has great heuristic value as a means of developing complex hypotheses and in theory building (Kerlinger, 1986). An important objective in the present analysis was to examine plausible, multiple influences on performance in calculus. Calculus serves as an effective gate keeper for entry into the interesting and lucrative careers in science and technology; fields from which women and minorities have been traditionally excluded. Had sufficient numbers of former remedial students been among those who took the calculus course that provided us with our criterion measure, the path model would have included remedial mathematics as an intermediate variable between HSGPA and SATMATH and the criterion variable, CALCTOT.

If the intent of remediation in mathematics, at the university level, is to prepare students for more advanced study, one might ask, "Why not simply examine whether students who successfully complete remedial courses are successful in subsequent courses?" The question is legitimate, but neglects to take into account the substantial nature of the mathematics deficiencies with which many students enter the University, and the influences that are associated with those deficiencies. The analysis model we propose takes these background variables into account. The question we would ask is, "What is the effect of remedial instruction on performance in subsequent mathematics courses, after controlling for the effects of variables such as ethnicity, sex, high school academic record, and college entrance scores?" Our hypothesis would be that, with these variables taken into account, the direct path from remedial math to the final dependent variable (CALCTOT in the present model) would not be significantly different from zero. Such information supplements, and places in a larger theoretical context, the important but simpler question of whether or not remedial courses are effective.

Several conditions would have to be met in order to apply this kind of analysis plan with optimal effectiveness. In the present analysis we were fortunate in having examination scores from which to form the dependent variable. Data from the Registrar's office contains only a much less sensitive indicator of performance: pass or no pass. Ideally, if we are serious about wanting to evaluate remedial mathematics, or even nonremedial precursor courses (e.g., precalculus), each course in the sequence should include an agreed-upon pool of test items that are common to the final examinations of all sections of the same course.

The entire question of examining the influence of remediation on performance in subsequent courses may be moot if the pattern found with these data is typical. We were very surprised that so few of the calculus students had histories of remedial mathematics instruction. Since remedial courses do not bear credit at the University of California, we assumed that preparation for more advanced courses would constitute a major motivation for taking remedial work. It is possible that students initially take remedial courses with that intent but then become discouraged with the possibility of pursuing mathematics. Plans may therefore be shifted to paths that do not require mathematics. On the other hand, it is possible that many students accomplish their objectives by taking remedial mathematics. For the majority it may suffice, by preparing them to select from a variety of courses outside mathematics that meet the quantitative requirement within the campus' general education framework. Whatever the case, it would be in the campus' interest to understand what is happening.

It seems clear, from the descriptive data presented, that the challenge of providing remedial instruction capable of preparing students for university level mathematics is not an easy one to meet. The average HSGPA of remedial students was below the norm for UC students, but within a range for which a successful college experience might be expected. On the other hand, the average mathematics SAT scores for the remedial students in our sample fell almost 100 points below the average for the non-remedial students, and 115 points below the 1984 average for University of California Freshmen (Report of the Work Group on Student Preparation and Remedial Education, 1986). If these scores reflect the degree of underpreparation of these students, it might not come as a great surprise if one or two remedial courses fail to produce students who are as ready as their better-prepared peers to tackle calculus.

The descriptive statistics also raise a question about the adequacy of precalculus as preparation for calculus. Of 38 students in Math 11A who had previously taken precalculus, only 2 failed precalculus. But 18 of these same students failed calculus. This seems to us to be a disproportionate number. Assuming that the students were equally motivated in the two classes, there are at least two alternative explanations. One possibility is that the expectations in this particular section of Math 11A were unrealistically high. A second explanation is that the content of precalculus may not be sufficiently well articulated to the concepts and skills required in calculus. If the goals of precalculus stand on their own, independent of the requirements of calculus, then this might be justified. We suspect, however, that most instructors think of the course primarily as a preparation for calculus. It would be worthwhile for the Mathematics Board to examine this issue.

Recommendations

Based on our findings, we would make the following recommendations:

1. All students entering the University, and who plan to take calculus, should be required to take the CSUC/UC Mathematics Diagnostic Examination. In the past, this examination has been optional at UCSC. We have learned that the Mathematics Board of Studies has already instituted this mandatory policy, which will take effect in the fall, 1986.

2. In addition to the examination mentioned above, all students planning to take calculus should meet with a mathematics advisor to discuss their previous mathematics experiences, the results of the diagnostic examination, and their mathematical objectives.

3. The Mathematics Board of Studies should take steps to ensure articulation of the objectives and content of remedial courses, precalculus, and calculus. This practice should be extended to include UCSC's primary feeder community colleges.

4. There should be an ongoing program of evaluation of the courses that constitute the path leading to calculus. Ideally, all sections of each course in this path should employ a common set of test items as part of the final examination. The provision for a common set of items, agreed to by all of the instructors, should, in addition to remedial courses and precalculus, also include the calculus course. This would provide a criterion score for use in a path analytic model.

5. Instructors of remedial courses, precalculus, and calculus should make students aware of the support services that are available. This would include supplementary materials, such as video tapes available in the Mathematics/CIS Laboratory, free tutorial help, and support services available through the Reentry Program and EOP/SAA.

References

Dweck, C. S. (1975). The role of expectations and attributions in the alleviation of learned helplessness. Journal of Personality and Social Psychology, 7, 674-685.

Kerlinger, F. N. (1986). Foundations of behavioral research (3rd ed.). New York: Holt, Rinehart & Winston.

Henderson, R. W. (1982). Personal and social causation in the school context. In J. Worell (Ed.), Psychological development in the elementary years. New York: Academic Press, pp. 359-407.

Henderson, R. W. (1986). Self-regulated learning: Implications for the design of instructional media. Contemporary Educational Psychology, 11, 405-427.

Jensen, A. R. (1980). Bias in mental testing. New York: Free Press.

Report of the Work Group on Student Preparation and Remedial Education. (1986). Berkeley: Office of the Senior Vice President - Academic Affairs, University of California.

Footnotes

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APPENDICES

MATH LEARNING SURVEY

Please complete the following questionnaire as accurately as you can. There are no right or wrong answers. It is important to respond to each item. If you are uncertain about your response, mark the statement that comes closest to your opinion. The frame of reference for every item is mathematics, even if it doesn't specifically say so.

Each item consists of two contrasting statements, separated by the word "but" for purposes of clarity. For each item, choose the statement that is most true for you. Then place an "X" on the blank indicating whether that statement is "really" true for you, or "sort of" true for you.

Please make only one response for each item.

Please write your name and your student identification number below. This information will be discarded as soon as data have been entered for analysis. All data are completly confidential. Individuals will not be identified in any analyses or reports.

NAME:________________________________ STUDENT ID #:________________________.

Sample Item

	Really true for me	Sort of true for me				Sort of true for me	Really true for me
(a)	-------	X -------	Some people enjoyed taking calculus	BUT	Some people hated taking calculus	------	------

A person who mildly, or "sort of" enjoyed taking calculus would mark this item as shown above.

1.	-----	-----	Some people never miss math classes	BUT	Some people skip math classes frequently	-----	-----
2.	-----	-----	Some people use the math tutors	BUT	Some people do not use the math tutors	-----	-----
3.	-----	-----	Some people read every assigned chapter in the math text	BUT	Some people use the math text only for reference	-----	-----

	Really true for me	Sort of true for me				Sort of true for me	Really true for me
4.	-----	-----	Some people do all assigned homework in math.	BUT	Some people do very little of the assigned math homework	-----	-----
5.	-- --	-----	Some people study math with classmates	BUT	Some people study math alone	-----	-----
6.	-----	-----	Some people think math texts skip important information	BUT	Some people think math texts give all the necessary information for solving the problems	-----	-----
7.	-----	-----	Some people only study math before tests	BUT	Some people review math almost every day	-----	-----
8.	-----	-----	Some people have a natural ability to learn math	BUT	Some people can never expect to learn math well	-----	-----
9.	-----	-----	Some students feel intimidated by symbols and math notation	BUT	Some students feel comfortable using math symbols and notation	-----	-----
10.	-----	-----	Some students think math teachers give adequate explanations	BUT	Some students think math teachers skip important steps in their explanations	-----	-----
11.	-----	-----	Some people study math as soon after class as possible	BUT	Some people put off studying math as long as possible	-----	-----
12.	-----	-----	Some people give up and quit when they don't understand a problem	BUT	Some people call someone for help when they don't under-stand a problem	-----	-----
13.	-----	-----	Some people learn best by hearing an explanation	BUT	Some people learn best by seeing an example	-----	-----
14.	-----	-----	Some people use big blocks of time for mathematics study	BUT	Some people study math for short periods of time	-----	-----

	Really true for me	Sort of true for me				Sort of true for me	Really true for me
15.	-----	-----	Some people find the wording of math tests hard to understand	BUT	Some people understand the wording of math tests easily	-----	-----
16.	-----	-----	Some people feel that math textbooks skip important steps in their explanations	BUT	Some people find the explanations in the text are adequate	-----	-----
17.	-----	-----	Some people do well in math even if they don't spend much time studying	BUT	Some people must spend a lot of time studying to do well in math	-----	-----
18.	-----	-----	Some students do not work through the examples in the text	BUT	Some students do work through the examples in the text, even if they aren't assigned	-----	-----
19.	-----	-----	Some students can visualize and explain abstract concepts	BUT	Some students have trouble visualizing and explaining abstract concepts	-----	-----
20.	-----	-----	Some students learn best from a teacher of the same sex	BUT	For some students, the sex of the teacher is irrelevant to learning	-----	-----
21.	-----	-----	Some students can understand a peer's explanations better than the teachers'	BUT	Some students find teachers' explanations easier to follow.	-----	-----
22.	-----	-----	Some people never use their notes to study for a test.	BUT	Some people use their notes to study for tests	-----	-----
23.	-----	-----	For some students, the etnic background of a teacher is irrelevant	BUT	Some students feel more comfortable learning from a teacher of their own ethnicity	-----	-----
24.	-----	-----	Some students start work on a math problem by trying to represent it in a sketch	BUT	Some students start work on a math problem by trying to use a formula	-----	-----

	Really true for me	Sort of true for me				Sort of true for me	Really true for me
25.	-----	-----	When some students do poorly on a test they feel they lack math ability, and quit trying	BUT	When some students do poorly on a math test they begin to study harder	-----	-----

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Thank you for taking your time to complete this survey. If you would like to receive a summary of the results, write your name and summer mailing address below. If you prefer, you may get a copy when you return to school next fall. Just contact Ed Landesman, 359b, Applied Sciences Building, or Ron Henderson, 31 Merrill.

HAVE A GOOD SUMMER!

Appendix B

MATH LEARNING PROJECT

Subject's Name:____________________ ID:________________ Date:_____

I. Math History

- How did you feel about math as you were going through elementary and high school?
  - + If didn't like it, or had trouble
    - Can you remember when you first started having trouble (or disliking it)?
    - What happened that made you feel that way?
    - When did it happen? (grade?)
- What kind of high school did you go to?
  - Technical, general?
  - Urban, inner city?
  - ethnically/linguistically diverse?
- What math coursed did you take?
  - How did you do?
  - How well did it prepare you for math at the University?

II. Math Class Information

- What math classes did you plan to take when you came to the university?
  - Why did you decide on those particular classes?
  - How did you decide?
- What math courses have you taken at UCSC?
  - + If 1A or 1B not mentioned
    - Did you take any refresher courses that didn'c carry college credit?
      - Which one(s)?
- Do you remember anything in particular about the text you used?
  - Was it easy to understand?
  - Did it have all the information you needed?

III. Study Habits

- Tell me something about how you study for math/
  - Do you go to all your classes?
  - Do you study each day or during certain days only?
  - How do you deal with frustration when doing a math problem?
  - Do you usually study alone or with other students who are working on the same material?
  - Are you involved in extra-curricular activities? Or a job?
    - How do they affect your studying?

IV. Math Self-Efficacy

- Would you say that math has come easily to you, or have you had to struggle with it?
- Do you like doing math?
- Did anyone ever tell you that you just couldn't (or shouldn't) take math?
  - Tell me more about that.

V. Other

- How could your UCSC math preparation for calculus have been improved?
- Some UC campuses are thinking of not offering any math courses prior to precalcusus. They would handle it by having students take the courses from a Community College. Community College instructors would come onto the UC campus to give the course. How do you feel about that?
  - How do you think students who are admitted to UC, but who need such instruction, would feel about this?

UNIVERSITY OF CALIFORNIA, SANTA CRUZ

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MERRILL COLLEGE SANTA CRUZ, CALIFORNIA 95064

May 30, 1986

Dear

We are conducting a survey of attitudes toward mathematics, and of the various approaches and support systems students employ in their study of mathematics. As a student who has been enrolled in one or more mathematics courses on this campus, you have been selected to participate in this survey.

We realize this is a busy time of the year, but the information you will provide by participating is very important. The University of California is very interested in determining the conditions necessary to ensure that students can be successful in their study of mathematics. The present survey provides information relevant to one aspect of that effort.

Of course, you are under no obligation to participate, and you may decline without any prejudice whatsoever. However, we do hope you will decide to give us a few minutes of your time by completing the enclosed survey and returning it to us in the envelope provided. Please respond to the items in the questionnaire as honestly as you can. There are no "best" or preferred answers. and you have our assurance that your identity will not be revealed. All data will be pooled for analysis, and no individual names will be used in reporting.

There are no risks or discomforts associated with participation in this survey.

Please sign the enclosed "Consent Form" and return it with your completed survey.

Thank you for your cooperation. If you would like to have a summary of the results of the survey, please let us know by providing the information requested at the end of the survey. Or you may request one when you return next fall by contacting one of us. Ed Landesman's office is in room 359b Applied Sciences, and Ron Henderson is in 31 Merrill College.

Sincerely,

Edward M. Landesman
Professor of Mathematics

Ronald W. Henderson
Professor, Education and Psychology

Appendix D

Response Frequencies for Math Learning Survey

Statement		Statement	
Really true for me	Sort of true for me	Sort of true for me	Really true for me
Frequencies			
Some people skip math classes frequently		Some people never miss math classes	
0	2	10	11
Some people use the math tutors		Some people do not use the math tutors	
10	4	4	5
Some people use the math text only for reference		Some people read every assigned chapter in the math text	
2	4	8	8
Some people do very little of the assigned math homework		Some people do all assigned homework in math	
1	1	7	14
Some people study math alone		Some people study math with classmates	
13	5	4	1
Some people think math texts skip important information		Some people think math texts give all the necessary information	
4	4	14	1
Some people only study math before tests		Some people review math almost every day	
7	6	9	1
Some people can never expect to learn math well		Some people have a natural ability to learn math	
3	5	11	4

Appendix D, Continued

	Really true for me	Sort of true for me		Sort of true for me	Really true for me
Some students feel intimidated by symbols and math notation	3	10	Some students feel comfortable using math symbols and notation	7	3
Some students think math teachers skip important steps in their explanations	2	4	Some students think math teachers give adequate explanations	7	9
Some people put off studying math as long as possible	7	10	Some people study math as soon after class as possible	3	3
Some people give up and quit when they don't understand a problem	2	2	Some people call someone for help when they don't understand a problem	8	11
Some people learn best by hearing an explanation	1	0	Some people learn best by seeing an example	1	19
Some people study math for short periods of time	1	4	Some people use big blocks of time for mathematics study	4	11
Some people find the wording math tests hard to understand	5	5	Some people understand the of wording of math tests easily	7	4
Some people feel that math textbooks skip important steps in their explanations	4	6	Some people find the explanations inthe text are adequate	8	3

Appendix D, Continued

Statement	Really true for me	Sort of true for me	Sort of true for me	Really true for me	Statement
Some people must spend a lot of time studying to do well in math	11	6	5	1	Some people do well in math even if they don't spend much time studying
Some students do not work the examples in the text	1	4	8	8	Some students do work through the examples in the text, even if they aren't assigned
Some students have trouble visualizing and explaining	5	6	9	2	Some students can visualize and explain abstract concepts
Some students learn best from a teacher of the same sex	1	1	2	18	For some students, the sex of the teacher is irrelevant to learning
Some students can understand a peer's explanations better than the teachers	4	6	8	2	Some students find teachers' explanations easier to follow
Some people never use their notes to study for a test	1	0	6	14	Some people use their notes to study for a test
Some students feel more comfortable learning from a teacher of their own ethnicity	0	1	5	17	For some students, the ethnic background of a teacher is irrelevant
Some students start work on a math problem by trying to use a formula	4	4	5	10	Some students start work on a math problem by trying to represent it in a sketch

Appendix D, Continued

Really true for me	Sort of true for me		Really true for me	Sort of true for me
When some students do poorly on a test they feel they lack math ability, and quit trying			When some students do poorly on a math test they begin to study harder	
2	4		11	6